Finding V_X

Reducing sawtooth climb data

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IN APRIL'S "TEST PILOT" WE turned the raw data from the sawtooth climb test flights in EAA's Young Eagles' RV-6A into useful preflight planning and cockpit-friendly in-flight tools. In the course of our data reduction we used engineering judgment to assess the quality of each data point, eliminated a suspected unreliable test point, and explained why we discarded it.

This month "Test Pilot" will use the same raw data to determine the RV-6A's maximum climb angle and the associated airspeed—V_X. Figure 1 looks like the data reduction worksheet for last month's 3,500-foot pressure altitude test block, but we've added columns for true airspeed and flight path angle. We'll use true airspeed in our V_X calculation, and the flight path angle illustrates a point we'll address later.

Fun with Numbers

We computed true airspeed using a complicated formula, but it's just as easy to use a chart or flight computer because you already know the density altitude and calibrated airspeed. For this example observed airspeed and calibrated airspeed are the same. This will probably not be true for your airplane.

By the time you perform your climb tests you should have completed your airspeed calibration tests and created a table that enables you to convert observed airspeed to calibrated airspeed. You can do this after flying your climb tests, so your worksheet should also have a calibrated airspeed column.

By plotting V_X versus density altitude we can have a single V_X reference for any altitude between the lowest and highest tested.

To be technically correct you would convert the calibrated airspeeds to equivalent airspeeds and use equivalent airspeed and density altitude for your true airspeed determination. You don't have to worry about the calibrated-to-equivalent conversion if your climb speeds and altitudes are below approximately 200 knots and 10,000 feet pressure altitude, respectively. That includes the RV-6A and just about every other single-engine propeller-driven airplane.

Using your calibrated airspeed and density altitude, determine the true airspeed for each of your sawtooth climbs and enter the results in the true airspeed column. You don't have to do this for data points you disregard using your engineering judgment, as we did for data point 2 in our worksheet. If that climb data was no good for V_Y determination, it won't be any good for V_X determination either.

Fun with Pictures

Figure 2 shows a climbing airplane with arrows representing its true air-

Test Order	Observed Airspeed	Start Press Alt	End Press Alt	Press Alt Block	Mid Press Alt	Elapsed Time	Avg ROC	OAT (deg F)	Density Alt	True Airspeed	Flight Path Angle	Remarks
1	80	3250	3750	500	3500	32	938	51	3794	85	6.28	
2	70	3250	3750	500	3500	37	811	51	3794			Low confidence. Wandered fast; explains faster ROC. Don't use.
3	90	3250	3750	500	3500	30	1000	51	3794	95	5.95	
4	65	3250	3750	500	3500	45	667	51	3794	69	5.49	VSI 650
5	100	3250	3750	500	3500	29	1034	52	3859	106	5.53	
6	75	3250	3750	500	3500	37	811	52	3859	79	5.78	VSI 750
7	85	3250	3750	500	3500	31	968	52	3859	90	6.09	VSI 1000
8	70	3250	3750	500	3500	40	750	51	3794	74	5.74	
9	95	3250	3750	500	3500	29	1034	52	3859	101	5.83	3/5 confidence
10	120	3250	3750	500	3500	30	1000	52	3859	127	4.46	VSI 1050
11	140	3250	3750	500	3500	39	769	52	3859	148	2.94	

Figure 1

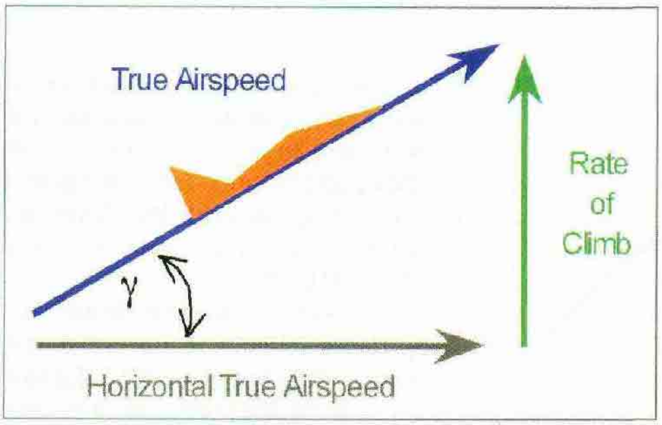

Figure 2

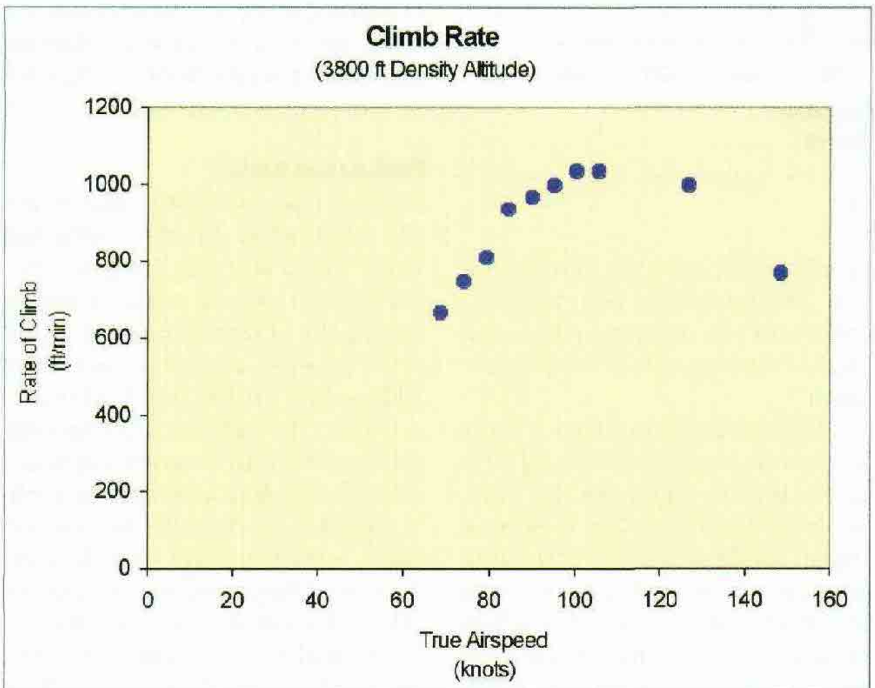

Figure 3

speed (which you just calculated) along its flight path and its vertical speed (which we calculated last month). We don't need the horizontal true airspeed component for our calculations, but this arrow illustrates the flight path or climb angle.

In April we showed how to create a working plot of *observed* airspeed versus climb rate for each of the three density altitudes tested. Figure 3 shows a similar plot, but this one uses data from our worksheet's *true* airspeed and climb rate columns. We must work with true airspeed during this portion of the data reduction; working with observed or calibrated airspeed results in either additional work or inaccurate V_X information.

Notice the axes in Figure 3 begin at zero, and the scales are linear (that is, the same distance between 20 and 40 knots as between 80 and 100 knots). These two criteria are essential for this method to work.

Looking at Figure 2 you can see that the longer the rate of climb arrow is in relation to the true airspeed arrow, the steeper the flight path angle will be. In other words, the steepest flight path angle occurs when the ratio of climb rate to true airspeed is the maximum attainable.

By fairing a curve through the data points in Figure 3 we get Figure 4, which shows us the missing data between the test points. Then we removed the data points to keep the plot from becoming too cluttered.

We can use Figure 4 to find the maximum climb-rate-to-true-airspeed ratio by drawing a straight line from the origin (0 airspeed, 0 climb rate) to the curve so it just touches the curve without passing through it. This tangent line from the origin to the curve touches the curve directly above V_X (in true airspeed) and directly to the right of V_X (in rate of climb). This tangent-to-the-curve relationship provides the most vertical speed for the least flight path speed, maximizing the climb-rate-to-true-airspeed ratio.

More Fun with Numbers

Now that we know the airplane's true airspeed V_X and the climb rate when flown at this airspeed, we can use the climb triangle in Figure 2 and a little trigonometry to calculate the V_X climb angle.

The three speed arrows in Figure 2 depict a right triangle, which allows us to use the simple sine (SIN) formula to determine the flight path angle (γ).

$$\mathbf{SIN}(\gamma) = \frac{\textbf{length of opposite side}}{\textbf{length of hypotenuse}}$$

$$\mathbf{SIN}(\gamma) = \frac{\textbf{rate of climb}}{\textbf{true airspeed}}$$

Our true airspeed is in knots, and the rate of climb is in feet per minute, so we'll have to apply a conversion to ensure the distances and times are in the same units. Using the V_X climb rate and true airspeed V_X from Figure 4, we have

$$\mathbf{SIN}(\gamma) = \frac{\textbf{910 ft/min}}{\textbf{84 nm/hr}} \times \frac{\textbf{60 min/hr}}{\textbf{6076 ft/nm}}$$

The 60 minutes per hour and the 6,076 feet per nautical mile are simple conversions to make the units cancel, so we're left with a number with no units. If your data are in miles per hour instead of knots, replace the 6,076 with 5,280. Once you know the SIN of the flight path angle, you can use a calculator or trig table to determine the flight path angle. For our example, the

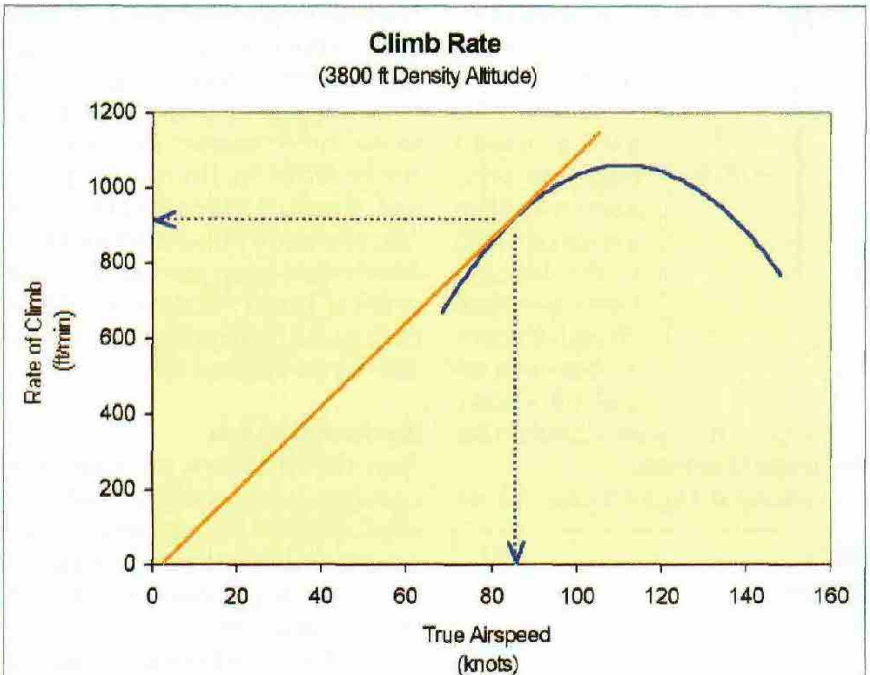

Figure 4

flight path angle is 6.14 degrees.

We've said this is the steepest flight path angle attainable, but our worksheet shows a slightly steeper angle in the first data row. The reason for the apparent discrepancy is this data point lies slightly above the curve we fitted to the data points. (Notice the fourth data point from the left in Figure 3.)

We're going to accept the lower climb angle determined by the graphical method for three reasons. First, we have more confidence in the curve because of how well it "fits" all the other data points. Second, the lower climb angle is more conservative for preflight planning. Third, the two V_X values differ by only 1 knot.

Next we convert the V_X true airspeed to a V_X observed airspeed so we can use it in the cockpit. We determined a true airspeed of 84 knots from the graphical analysis of our 3,800-foot density altitude plot. Using a flight computer this becomes 79 knots for our RV-6A's observed V_X airspeed. You would convert your true airspeed to calibrated airspeed and then use the plot or table you created during your airspeed calibration to determine your airplane's corresponding observed airspeed.

The RV-6A does not have a climb angle indicator, and working with angles is more difficult than working with distances when determining an airplane's takeoff performance on any given day. Converting the climb angle to a climb gradient makes it easier to determine how much horizontal distance the V_X climb segment will need to clear an obstacle near the runway's end. A climb gradient is simply how much altitude the airplane gains for a given horizontal distance traveled. We already know the climb angle for our V_X climb, so we can use the trigonometric tangent (TAN) function to find the climb gradient.

$$\mathrm{TAN}(\gamma) = \frac{\text{length of opposite side}}{\text{length of adjacent side}}$$

$$\mathrm{TAN}(\gamma) = \frac{\text{altitude gained}}{\text{horizontal distance traveled}}$$

Once you know the climb angle for a particular density altitude, all you have to do is find the TAN of that angle to know the climb gradient. For our example the climb angle is 6.14 degrees, and the TAN (6.14) is 0.108.

To find the horizontal distance required to clear a 50-foot obstacle when climbing at V_X with a density altitude of 3,800 feet, we'd divide the obstacle height by the climb gradient (50/0.108 = 463).

Remember that this distance does not account for the takeoff roll or the airborne distance used while accelerating to V_X. It also does not account for wind or raising the flaps or any other configuration change, so be sure to plan conservatively.

Problems with Numbers

Now we repeat the entire process for the other tested altitudes. Using the three tested altitudes from our RV-6A sawtooth climb tests, we determined the observed airspeed V_X to be 79 knots for all three density altitudes—3,800, 6,700, and 9,600 feet.

Hmm. Theoretically, the observed airspeed should increase approximately one-half percent for each 1,000 feet of altitude. Before we throw out all our hard work, let's see how this disagreement with theory affects the usefulness of our data.

Typically, V_X climbs are performed for short durations to clear an obstacle, so we're not worried about holding the exact climb airspeed for an extended time. But we do want to make sure that flying a couple of knots slower or faster than V_X does not seriously affect the climb angle.

We analyzed how flying 2 knots faster and slower than the V_X airspeeds from our test results would affect climb angle and horizontal distance to clear a 50-foot obstacle. The largest climb angle effect occurs at the 3,800-foot density altitude, and it's 0.08 degrees.

This means if we climbed the RV-

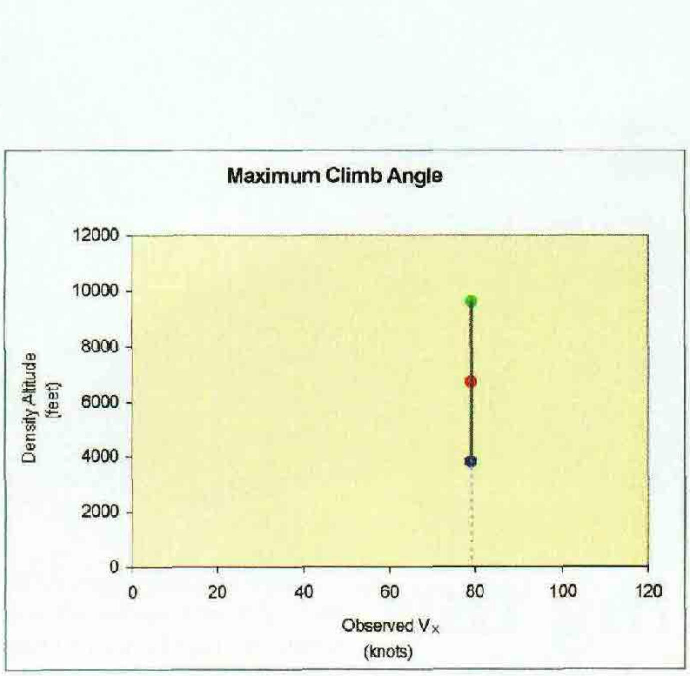

Figure 5

6A 2 knots faster than the 79 knots indicated by our test results, we would require an additional 6 feet of horizontal distance to clear that 50-foot obstacle. If we flew 2 knots slower, we'd require an additional 4 feet of horizontal distance. At the 9,700-foot density altitude the penalty for flying 2 knots faster or slower than V_X is 1 foot of additional horizontal distance to clear the 50-foot obstacle.

Projecting this analysis down to sea level, the additional horizontal distance should be no more than about 10 feet or about 1.4 percent of the horizontal distance from the beginning of the V_X climb to the obstacle. We concluded that our test results are sufficiently accurate for the way we'll fly the airplane.

To this point we have V_X, the associated climb rate, and the climb gradient for our three tested density altitudes. By plotting V_X versus density altitude we can have a single V_X reference for any altitude between the lowest and highest tested. Figure 5 shows our plot. Because our data indicate the same 79-knot V_X observed airspeed, our line through the data is vertical.

Your results may be different and will probably result in a line that indicates a faster V_X for higher density altitudes. We've shown the line extrapolated to zero density altitude, but as we cautioned last month, this is a bit of a stretch. Another sawtooth climb test through a 1,500-foot density altitude should supply enough confidence to extrapolate this line to zero.

Over the past three months we introduced the sawtooth climb test technique, explained how to perform it, and shown how to reduce the data. We used raw data gathered from our test flights in the EAA's Young Eagles' RV-6A, and the resulting plots of V_Y versus density altitude and V_X versus density altitude can be used for preflight planning and in-flight reference. Our V_X versus density altitude data didn't exactly conform to theoretical predictions, but this allowed us to present a post-flight data analysis and, in this case, accept the data we acquired.

We weren't lazy during our climb tests. During each descent between climbs we gathered descent performance data, which will be next month's topic. Send your suggestions and comments to Test Pilot, EAA Publications, P.O. Box 3086, Oshkosh, WI 54903-3086 or *editorial@eaa.org* with TEST PILOT as the subject of your e-mail.